FORMS OF EQUILIBRIUM OF A RECTANGULAR PLATE IN GAS FLOW

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Investigation of the forms of equilibrium of an elongated rectangular plate in a gas flow, is connected with solving a nonlinear boundary value problem. The differential equation for normal displacements has the form [1, 2]

$$\frac{D}{h}\frac{d^4(w^*-w_0^*)}{dx^{*4}}+\sigma^*\frac{d^2w^*}{dx^{*2}}-\frac{p_0^*}{h}\left[1-\left(1+\frac{\varkappa-1}{2}M\frac{dw^*}{dx^*}\right)^k\right]=0 \tag{1}$$

where

$$k=\frac{2\varkappa}{\varkappa-1}$$

The stress σ^* in the middle surface of the plate can be found from the following integral equation

$$\sigma^*=\frac{E}{2c(1-\nu^2)}\int_0^c\left[\left(\frac{dw_0^*}{dx^*}\right)^2-\left(\frac{dw^*}{dx^*}\right)^2\right]dx^* \tag{2}$$

Following [2] we shall assume that $\varkappa = 1.4$ and introduce the following numerical parameters

$$w=\frac{w^*}{h},\quad \sigma=12(1-\nu^2)\frac{\sigma^*c^2}{Eh^2},\quad x=\frac{x^*}{c}$$
$$q=12(1-\nu^2)\frac{p_0^*c^4}{Eh^4},\quad \eta=M\frac{h}{c} \tag{3}$$

After a series of manipulations, with (3) taken into account, we obtain from (1) and (2),

$$w''''+\sigma w''+qw'\left[1+\frac{3}{5}\eta w'+\frac{1}{5}(\eta w')^2+\frac{1}{25}(\eta w')^3+\frac{3}{625}(\eta w')^4+\frac{1}{3125}(\eta w')^5+\right.$$
$$\left.+\frac{1}{109375}(\eta w')^6\right]=w_0'''' \tag{4}$$

$$\sigma=6\int_0^1(w_0'^2-w'^2)\,dx \tag{5}$$

Here and in the following, differentiation with respect to x will be denoted by a prime. Let us assume that the form of initial imperfection of the plate is described by

$$w_0=w_{00}\sin\pi x \tag{6}$$

We shall adopt the value $w_{00} = 5$ in our computations in order to be able to correlate them with the results obtained by A. Iu. Birkhan who used the method of finite differences [2] for a plate with hinged edges.

Inserting (6) into (4) and (5), we obtain the following expressions:

$$w''''+\sigma w''+qw'\left[1+\frac{3}{5}\eta w'+\frac{1}{5}(\eta w')^2+\frac{1}{25}(\eta w')^3+\frac{3}{625}(\eta w')^4+\frac{1}{3125}(\eta w')^5+\right.$$
$$\left.+\frac{1}{109375}(\eta w')^6\right]=\pi^4w_{00}\sin\pi x \tag{7}$$

$$\sigma=3\pi^2w_{00}^2-6\int_0^1 w'^2dx \tag{8}$$

To solve the problem we must replace the integral condition (8) with a differential

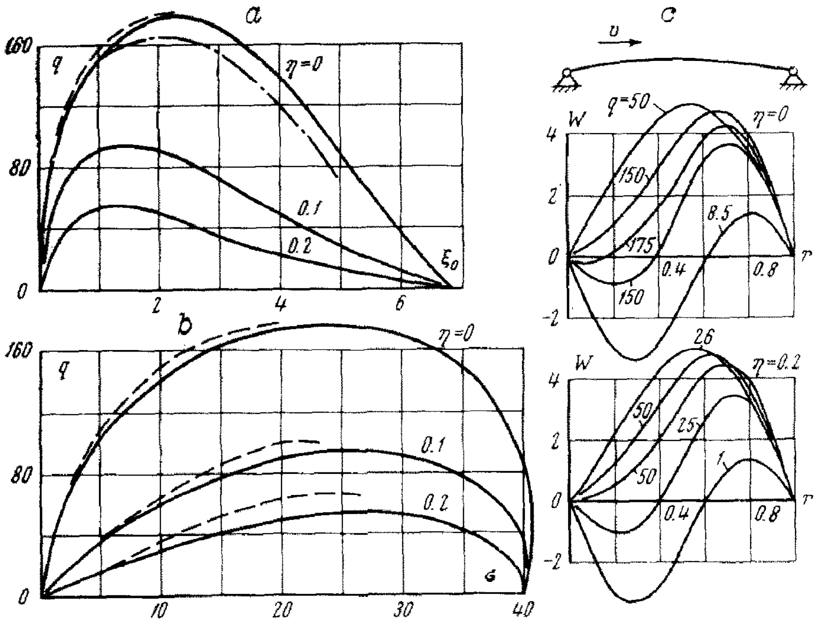

Fig. 1

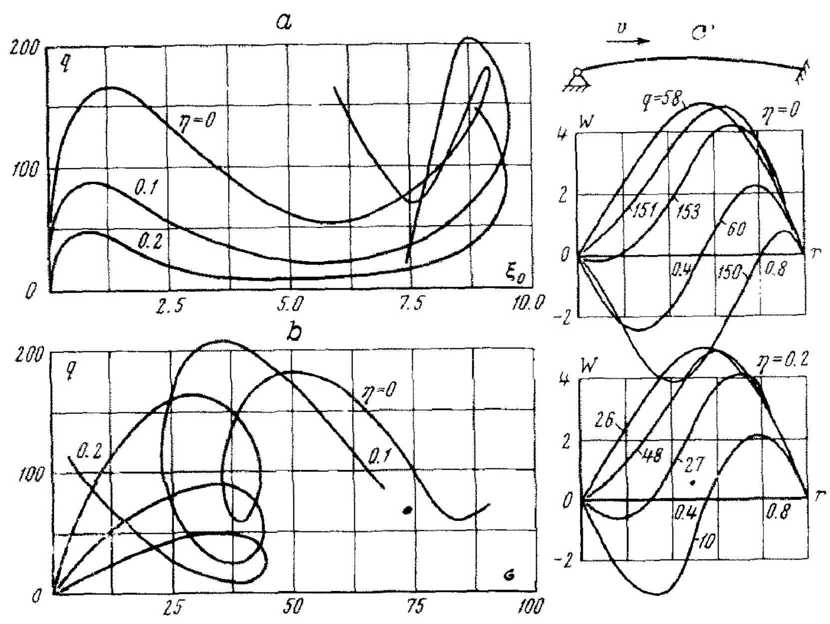

Fig. 2

one [3], assuming that the upper limit of integration is variable and introducing the following auxiliary function:

$$\lambda(x) = 3\pi^2 w_{00}^2 - 6\int_0^x w'^2 dx - \sigma$$

This yields

$$\lambda'(x) = -6w'^2, \quad \lambda(0) = 3\pi^2 w^2_{00} - \sigma, \quad \lambda(1) = 0 \tag{9}$$

The following expressions for the bending moment and the angle of rotation of the normal to the middle surface of the plate, are necessary for the boundary conditions to hold

$$M_1 = 12(1-\nu^2)\frac{M_1^* c^2}{Eh^4} = -(w'' - w_0''), \quad \theta = \frac{c\theta^*}{h} = w' - w_0' \tag{10}$$

Taking into account (6), we obtain

$$M_1 = -w'' - w_{00}\pi^2 \sin \pi x, \quad \theta = w' - w_{00}\pi \cos \pi x \tag{11}$$

Various methods of clamping the plate edges were considered, and are given below.

1. Plate hinged along all edges (Fig. 1)

$$x = 0, \quad w = w'' = 0, \quad \lambda = 3\pi^2 w_{00}^2 - \sigma \tag{12}$$
$$x = 1, \quad w = w'' = 0, \quad \lambda = 0$$

2. Plate hinged along the left edge and rigidly clamped along the right edge (Fig. 2)

$$x = 0, \ w = w'' = 0, \ \lambda = 3\pi^2 w_{00}^2 - \sigma \tag{13}$$
$$x = 1, \ w = \theta = 0, \ \lambda = 0$$

3. Plate rigidly clamped along the left edge and hinged along the right edge (Fig. 3)

$$x = 0, \quad w = \theta = 0, \quad \lambda = 3\pi^2 w_{00}^2 - \sigma \tag{14}$$
$$x = 1, \ w = w'' = 0, \ \lambda = 0$$

4. Plate rigidly clamped along all edges (Fig. 4)

$$x = 0, \quad w = \theta = 0, \quad \lambda = 3\pi^2 w_{00}^2 - \sigma \tag{15}$$
$$x = 1, \quad w = \theta = 0, \ \lambda = 0$$

All problems were solved using the numerical algorithm given in [3]. Let us consider the first problem in more detail.

Let us fix the parameter q at some value $q = q_0$ and assume arbitrary initial values for the function and for its derivatives, taking however (12) into account

$$x = 0, \ w = w'' = 0, \ w' = \alpha_1, \ w''' = \alpha_2, \ \sigma = \alpha_3, \ \lambda = 3\pi^2 w_{00}^2 - \alpha_3 \tag{16}$$

We then integrate the system (7)–(9) numerically and determine the following expressions

$$w(1) = w(\alpha_1, \ \alpha_2, \ \alpha_3), \quad w''(1) = w''(\alpha_1, \ \alpha_2, \ \alpha_3) \tag{17}$$
$$\lambda(1) = \lambda(\alpha_1, \ \alpha_2, \ \alpha_3)$$

Solution of the problem can be obtained under the condition that α_i are chosen so as to fulfil the conditions

$$w(\alpha_1, \ \alpha_2, \ \alpha_3) = 0, \quad w''(\alpha_1, \ \alpha_2, \ \alpha_3) = 0, \ \lambda(\alpha_1, \ \alpha_2, \ \alpha_3) = 0 \tag{18}$$

Thus we have reduced the problem to a nonlinear algebraic system (18), in a manner similar to that employed in [3]. In the obtained algorithm particular attention was given to providing the conditions of solvability of the system of nonlinear equations. This makes it possible to solve problems, in which the parameter varies nonmonotonously [3].

Since the possibility of obtaining a solution depends largely on how nearly the required solution is approached by the initial approximation, the numerical algorithm is constructed in such a manner, that the initial approximation can be predicted with any predetermined degree of accuracy [3].

Let the solution of the problem be known for some fixed value $q = q_0$ of the parameter, defined in terms of the following values of the initial parameters: $\alpha_1^\circ, \alpha_2^\circ$ and α_3°. Let us now increase the value of q to $q = q_0 + \Delta q$ and write the initial approximation

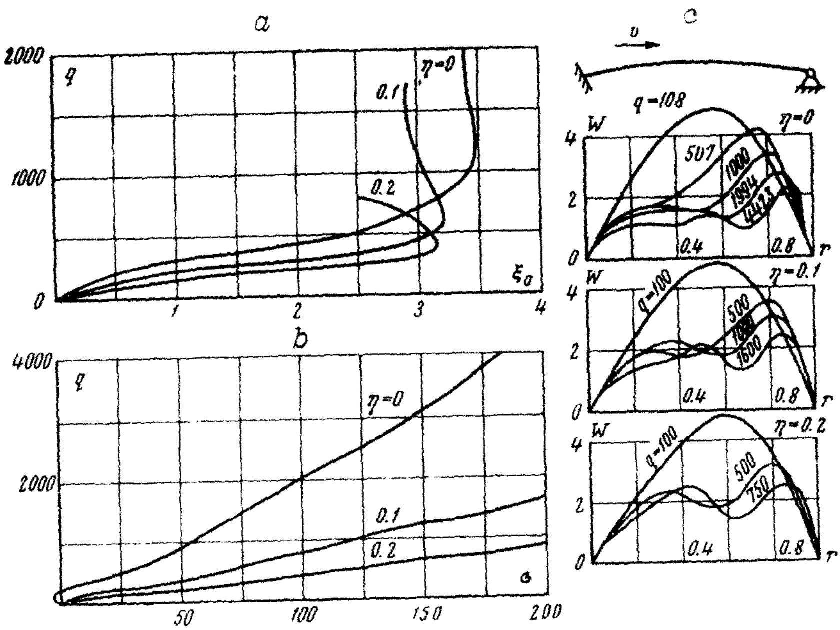

Fig. 3

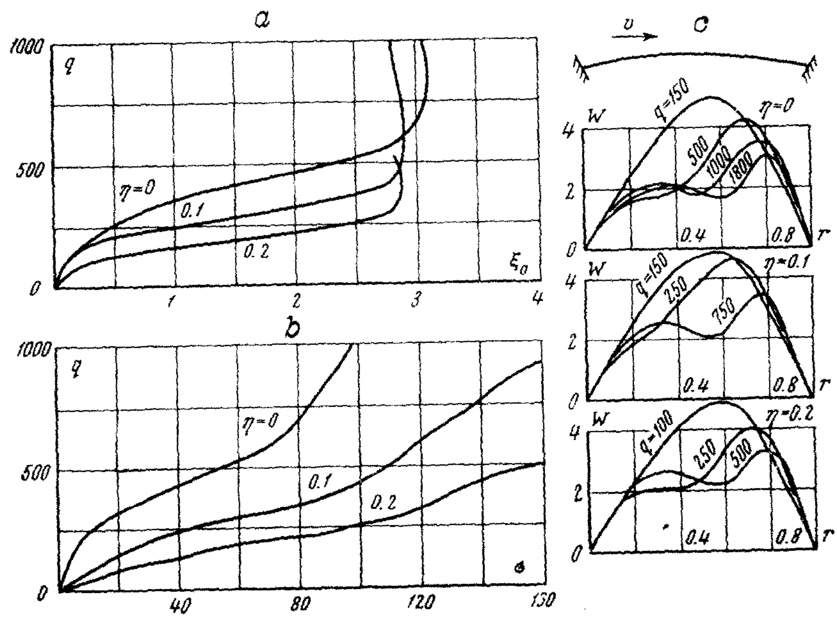

Fig. 4

in the form

$$\alpha_i = \alpha_i^\circ + \frac{d\alpha_i}{dq}\Delta q + \frac{1}{2}\frac{d^2\alpha_i}{dq^2}\Delta q^2 \quad (i = 1,\ 2,\ 3) \tag{19}$$

Let the process of solving the system (18) converge after k_0 iterations. If $k_0 > 2k$, **then we halve Δq**, estimate the initial approximation using formulas (19) and repeat the process. We continue to do this, until the condition $k \leqslant k_0 \leqslant 2k$ is met. If $k_0 < k$, we double the interval in the next step. When applying the modified Newton's method the values of k were taken within 3 and 4.

Approximate values of the derivatives in (19) are obtained by means of the difference formulas. In the first step we take into account the first terms only, first two terms in the second step, and the remaining terms in the following steps. Certain problems were solved without using (19). Instead, α_i were computed by extrapolating them into a polynomial.

Parameter q may be found to vary nonmonotonously during the computing process. For this reason, the behavior of the first derivatives appearing in (19) must be carefully observed when moving along the curve corresponding to the solution. Should the modulus of any of them exceed unity, then the corresponding initial value of the unknown function must be used as a parameter in the next steps. If, e.g. $|d\alpha_1 / dq| > 1$, then we use α_1 as the parameter and refine q in the process of solving the nonlinear system. The initial approximations are predicted here using formulas analogous to (19).

With the algorithm constructed in this manner we may advance along the curve corresponding to the solution, and satisfy the condition of single-valuedness in the neighborhood of the fixed parameter. This also yields the condition of solvability of the system of nonlinear equations and makes possible the application of the Newton's method as well as of its modification.

We use the solution of the problem at $q_0 = 0$ defined by the initial deformed state, to commence the computations. In addition we also have

$$\alpha_1^{\circ} = \pi w_{00}, \quad \alpha_2^{\circ} = -\pi^3 w_{00}, \quad \alpha_3^{\circ} = 0 \tag{20}$$

The system (8), (9) was integrated on the computer BESM-3M using the Runge-Kutta method with an automatic selection of the interval in x subject to condition that the loss of absolute accuracy on each interval does not exceed $\psi_1 = 10^{-6}$. Control runs were used to check the relative accuracy using the value $\psi_2 = 10^{-5}$. In both cases the results practically coincided.

Figures 1 to 4 depict the results of computations for various values of η, with ξ_0 denoting the additional bending at the center of the plate. Dash-dot line on Fig. 1 depicts the solution of the problem obtained by the Bubnov-Galerkin method with $\eta = 0$, the dotted line shows the results obtained by A. Iu. Birkhan [2] using the method of finite differences. Solution by the latter method was left incomplete, since the computational sequence diverged.

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